INTERNATIONAL CONFERENCE ON
COMPUTATIONAL METHODS IN
ENGINEERING & HEALTH SCIENCES 2023

Aerofoil Optimisation

A Report Submitted To the CFD Competition as a Part of the 6^{th}
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1 Acknowledgements

We would like to thank Dr. Manikandan Murugaiah for graciously accepting to be our mentor throughout the course of this competition. We would also like to thank Dr. Ping He for his prompt reply on the potential use of DaFOAM for this use case. Further, we would like to thank the Department of Aeronautical and Automobile Engineering at the Manipal Institute of Technology, Karnataka, India for providing us unrestricted access to their labs and the software used throughout the course of this project.

2 Abstract

To gain an insight into the problem parameters, a parametric sweep was conducted in Ansys Fluent. The design points to be optimised were identified using the results thus obtained. A genetic algorithm was coded and employed by coupling MATLAB and XFOIL. The optimised airfoil thus achieved was then subjected to a parametric sweep to verify the conformance to the problem parameters.

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5 List of Symbols and Abbreviations

Symbol	Description	Symbol	Description
CFD	Computational Fluid Dynamics	α	Angle of Attack
DaFOAM	Discrete Adjoint with OpenFOAM	MATLAB	Matrix Laboratory
y/c	Thickness to Chord Ratio	ρ	Density
a	Speed of Sound	U_∞	Freestream Velocity
M	Mach Number	M_∞	Freestream Mach Number
μ	Dynamic Viscosity	μ_∞	Freestream Dynamic Viscosity
CST	Class Shape Transformation	h	Altitude
t	Maximum Thickness	γ_{air}	Isentropic Expansion Factor
m	Position of Maximum Thickness	T	Temperature
p	Chordwise Position of Maximum Ordinate	P	Pressure
M_{air}	Molecular Mass of Air	K	Number of Aerofoils to be Generated in the First Iteration
AR	Aspect Ratio	C_n	nth CST Coefficient
P_n	nth PARSEC Coefficient	N	Number of Generations
C_L	Coefficient of Lift	C_D	Coefficient of Drag

6 Problem Description and Introduction

6.1 Problem Statement

The main aim of the project was to meet the constraints specified by the competition guidelines. Hence efforts were undertaken to optimise the NACA 4412 aerofoil for operation at 36,000 ft in the subsonic regime (below Mach 0.8). Further, several other constraints had to be met, namely,

* Minimum $^{L}/_{D}$ of 15
* AoA (α) between 2° and 5°
* Minimum C_l between 0.6 and 1.2
* Maximum thickness $^{y}/_{c} = 15\%$
* Manufacturing constraints

6.2 Introduction

A multi-faceted approach was taken to solving the given problem. A mesh validation study was conducted to verify the quality of the mesh by comparing the values found in the reference [1]. A parametric sweep was carried out in Ansys Fluent to evaluate the scope of the problem. Based on this, an optimisation using CST coefficients was conducted by integrating MATLAB with XFOIL. This was achieved using a genetic algorithm whose details are outlined in the following section. The optimised aerofoil thus achieved was subjected to a parametric sweep to validate and compile the resulting aerofoil's characteristics. The results were compiled, and a report was formulated using LaTeX.

7 Methodology

7.1 Aerofoil Characteristics According to Mach Number

Initially, the optimum aerofoil characteristics were studied according to Mach number, and the following study was conducted.

Table 7.1: Aerofoil Characteristics According to Mach Number

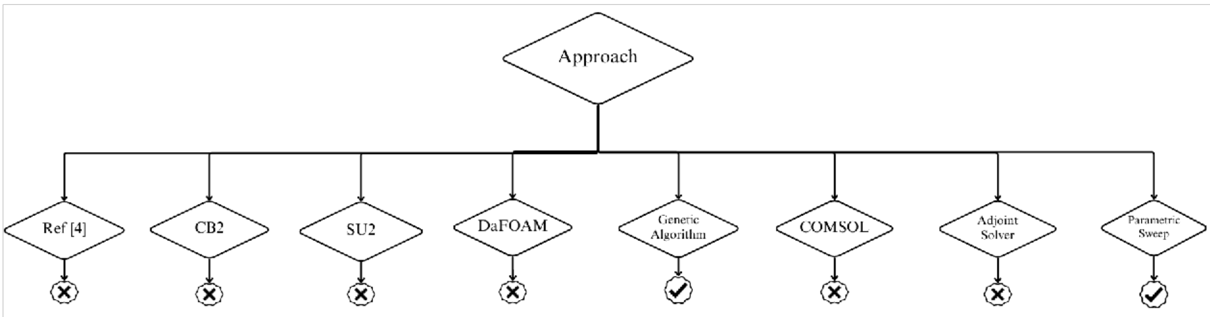
Mach Number	Characteristics
Mach 0.1 to Mach 0.5	At low Mach numbers (subsonic), aerofoil shapes are typically characterised by gentle curvature on the upper surface and a relatively flat lower surface. The camber (curvature) of the aerofoil provides lift while minimising drag. The thickness of the aerofoil may be relatively higher to generate more lift at low speeds.

Mach 0.6	Similar characteristics to Mach 0.5 but slightly thinner aerofoils are more efficient.
Mach 0.7	Transitional Mach number, aerofoils with a moderate thickness and slightly increased curvature on the upper surface work well.
Mach 0.8	Aerofoils designed for transonic flight, like supercritical aerofoils, become more relevant in this regime.

7.2 Approach

Several software were considered to optimise the aerofoil to meet the mission requirements. These included SU2, CB2, DaFOAM and COMSOL. These are outlined in the flow chart below. The OpenFOAM-based jacobian free discrete adjoint method (DaFOAM) was abandoned due to the fact that a shape constraint couldn't be implemented in the existing architecture of the solver. Dr Ping He, the original author of the solver, confirmed this. Similarly, CB2 and SU2 were also abandoned due to the shape constraint.

Figure 7.1: Outline of the Approach



7.3 Operating Conditions

The operating conditions for the study are as given below,

Table 7.2: Operating Conditions

Parameter	**Value**	**Parameter**	**Value**
h	36,000 ft	M_{air}	28.97 $^{kg}/_{mol}$
P_{air}	22729.31 Pa	ρ_{air}	0.36 $^{kg}/_{m^3}$
T_{air}	216.83 K	a_{∞}	295.19 $^{m}/_{s}$
μ_{∞}	1.825 x 10^{-5} Pa·S	γ_{air}	1.4

7.4 Aerofoil Generation

The NACA 4412 aerofoil was generated using the standard equations mentioned in reference [2]. The thickness distribution of the aerofoil is given by,

$$\pm y_t = \frac{t}{0.20}(0.29090\sqrt{x} - 0.12600x - 0.35160x^2 + 0.28430x^3 - 0.10150x^4) \tag{7.1}$$

Further, the mean camber line was obtained using the equations,

The forward of maximum ordinate is given by,

$$y_c = \frac{m}{p^2}(2px - x^2) \tag{7.2}$$

and the aft of maximum ordinate is given by,

$$y_c = \frac{m}{(1-p)^2}(1-2p) + 2px - x^2 \tag{7.3}$$

The slope of the above equations was used to obtain θ. This was then utilised to get the x and y coordinates of the upper and lower surfaces of the aerofoil using the following equations.

The upper surface of the aerofoil is given by,

$$x_u = x - y_t \cdot sin\theta \tag{7.4}$$

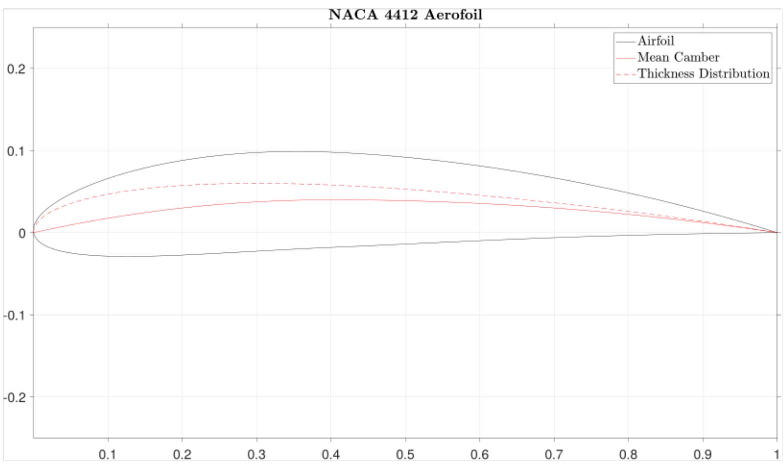
$$y_u = y_c + y_t \cdot cos\theta \tag{7.5}$$

The lower surface of the aerofoil is given by,

$$x_l = x + y_t \cdot sin\theta \tag{7.6}$$

$$y_l = y_c - y_t \cdot cos\theta \tag{7.7}$$

Figure 7.2: NACA 4412 Aerofoil



7.5 Mesh Validation Study and Parametric Sweep

A C-type mesh was created for the base aerofoil. The mesh Metrics are given below. A mesh validation study was conducted for $\alpha = 0$ and Reynolds number 5.9 x 10^6, using the k-ω SST turbulence model. The results obtained were compared with those outlined in reference [1], and an error of less than 5% was achieved (3.1%). After which, a parametric sweep was done by varying α and u_∞ in order to obtain a s comprehensive understanding of the problem parameters.

Figure 7.3: Generated Mesh

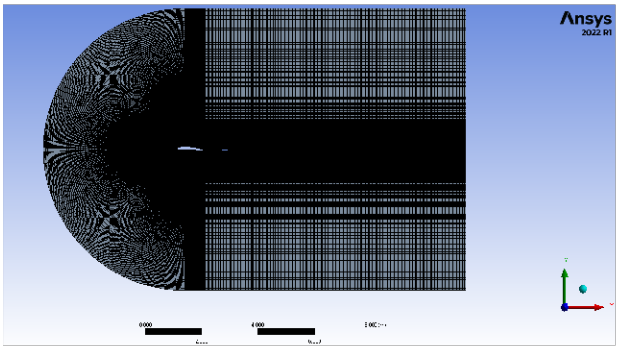


Table 7.3: Mesh Metrics

CST Coefficients	Value
Elements	409700
Average AR	8.5846
Maximum Skewness	0.5219
Minimum Orthogonality	0.4677

7.6 CST and PARSEC

The Class Shape Transformation (see reference [3]) and the PARSEC parametrisation techniques were utilised to optimise the aerofoil. The CST and PARSEC coefficients of the NACA 4412 aerofoil are as follows,

Figure 7.4: Class Shape Transformation

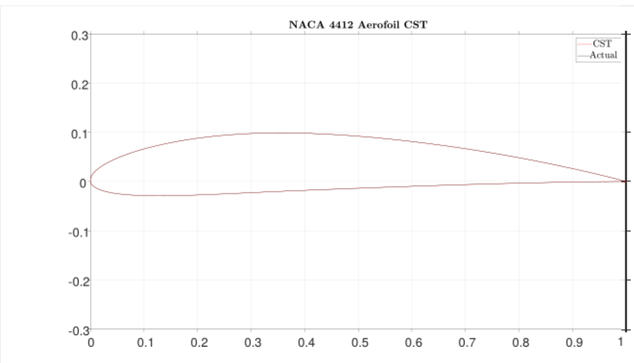


Figure 7.5: PARSEC

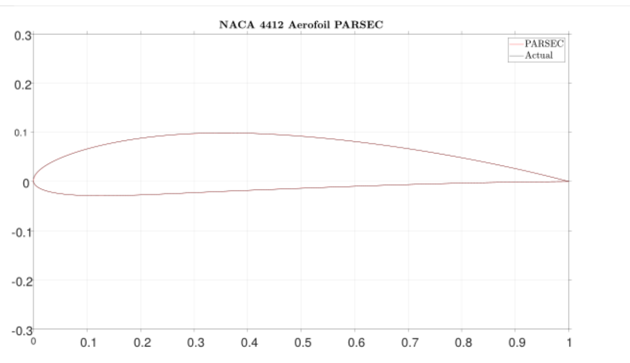


Figure 7.6: Class Shape Transformation

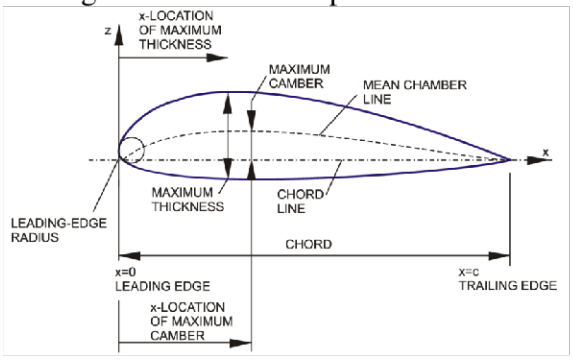


Image Source - Research Gate

Figure 7.7: PARSEC

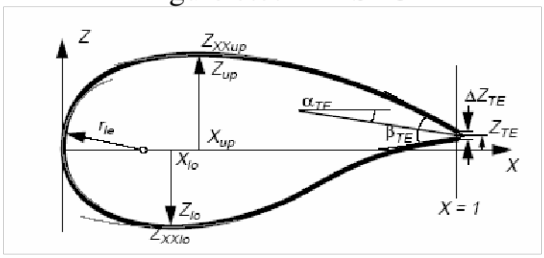


Image Source - Research Gate

Table 7.4: CST Coefficients

CST Coefficients	Value
C_1	0.2142
C_2	0.2881
C_3	0.2395
C_4	0.2881
C_5	-0.1328
C_6	-0.0112
C_7	-0.0469
C_8	0.0018

Table 7.5: PARSEC Coefficients

PARSEC Coefficients	Value	PARSEC Coefficients	Value
P_1	0.0230	P_7	0.0289
P_2	0.0092	P_8	0.8868
P_3	0.3589	P_9	-1.0116e^{-4}
P_4	0.0986	P_{10}	-8.9585e^{-5}
P_5	-0.7270	P_{11}	7.9002
P_6	0.1255	P_{12}	16.7892

7.7 Genetic Algorithm

A genetic algorithm was chosen due to its efficiency in finding the global optimum solution. It was also chosen due to its ability to solve complex problems and parallel capabilities.

Initially, the CST and PARSEC coefficients were varied according to the standard limits and limits approximated based on different speed regimes (refer to section 7.1). Based on these limits, an array of aerofoils were generated. These aerofoils were arranged in decreasing order of their L/D ratios. The fittest aerofoils were chosen based on their transcendence probabilities.

Based on the crossover and the mutation probabilities, the parameters were crossed and mutated. This process was carried out for multiple generations. Finally, the fittest aerofoil was then chosen. After running the genetic algorithm using both CST and PARSEC coefficients, it was decided to finalise the CST parametrisation technique due to its efficiency.

Figure 7.8: Flowchart

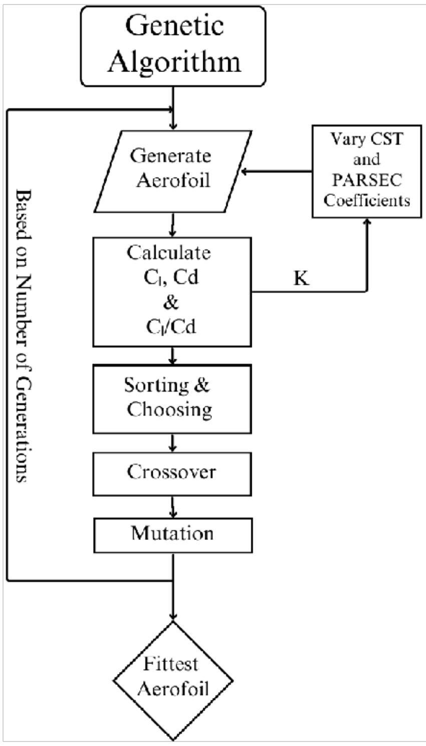


Table 7.6: Genetic Algorithm Specifications

Parameter	Value
K	30
Crossover Probability	75%
Transcendence Probability	5%
Mutation Probability	20%
Generations (N)	40

7.8 Verification

A C-type mesh was used for the verification of the optimised aerofoil. The mesh metrics are given below. A parametric analysis was carried out based on the k-ω SST turbulence model for the specified operating conditions.

Figure 7.9: Generated Mesh Using Optimised Aerofoil

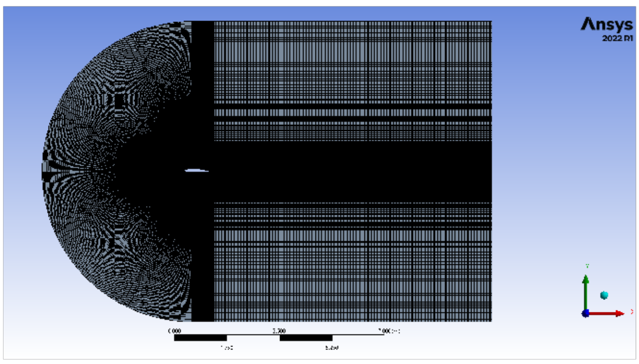


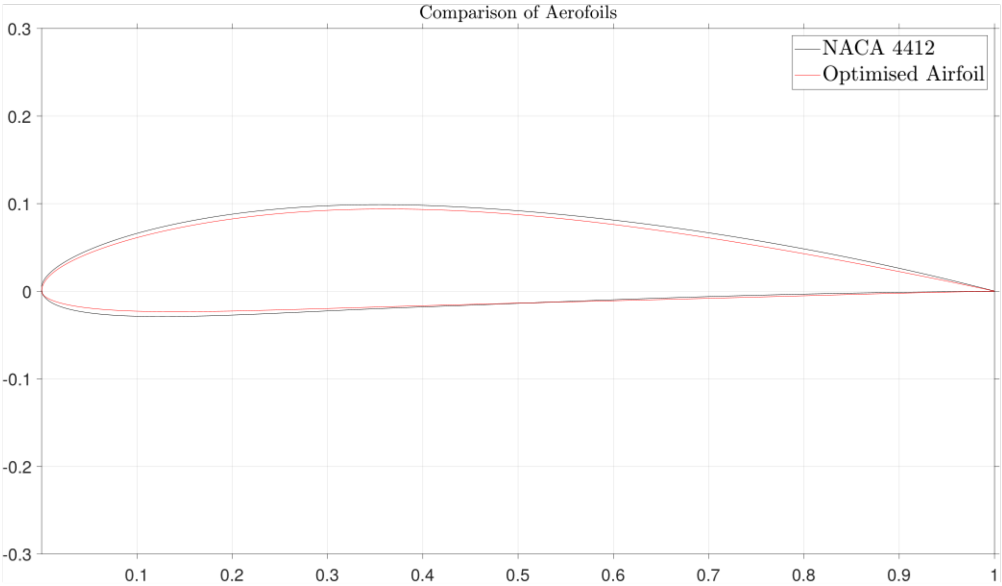
Table 7.7: Mesh Metrics

CST Coefficients	Value
Elements	409700
Average AR	8.6141
Maximum Skewness	0.6911
Minimum Orthogonality	0.4677

8 Results

The optimised aerofoil is plotted below,

Figure 8.1: Optimised Aerofoil



The following results were obtained after running a parametric sweep on the averaged angle of attack of 3.5° in Ansys Fluent.

Figure 8.2: Coefficient of Lift C_L

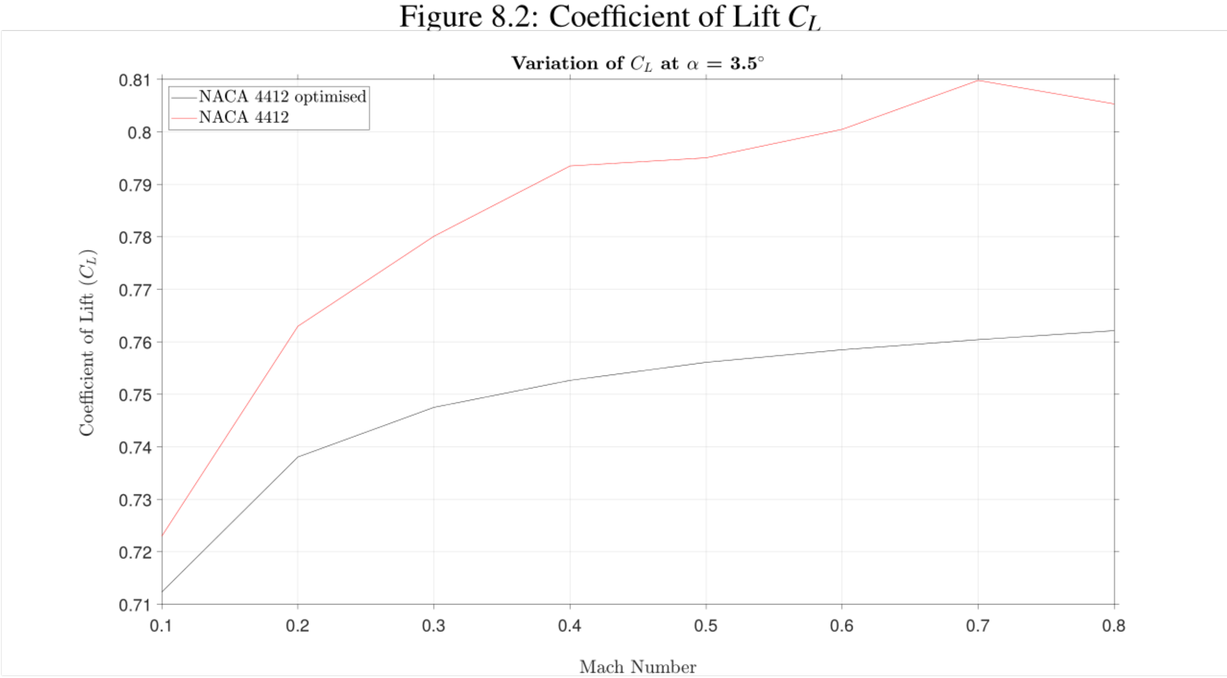


Figure 8.3: Coefficient of Drag C_D

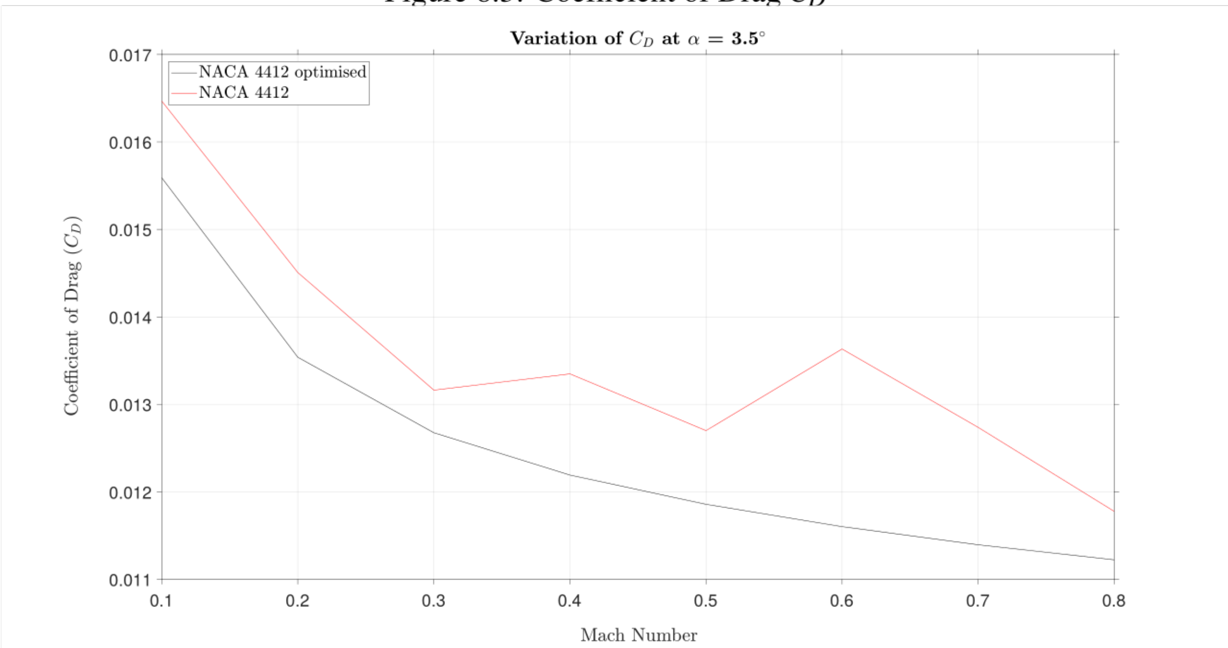


Figure 8.4: L/D Ratio

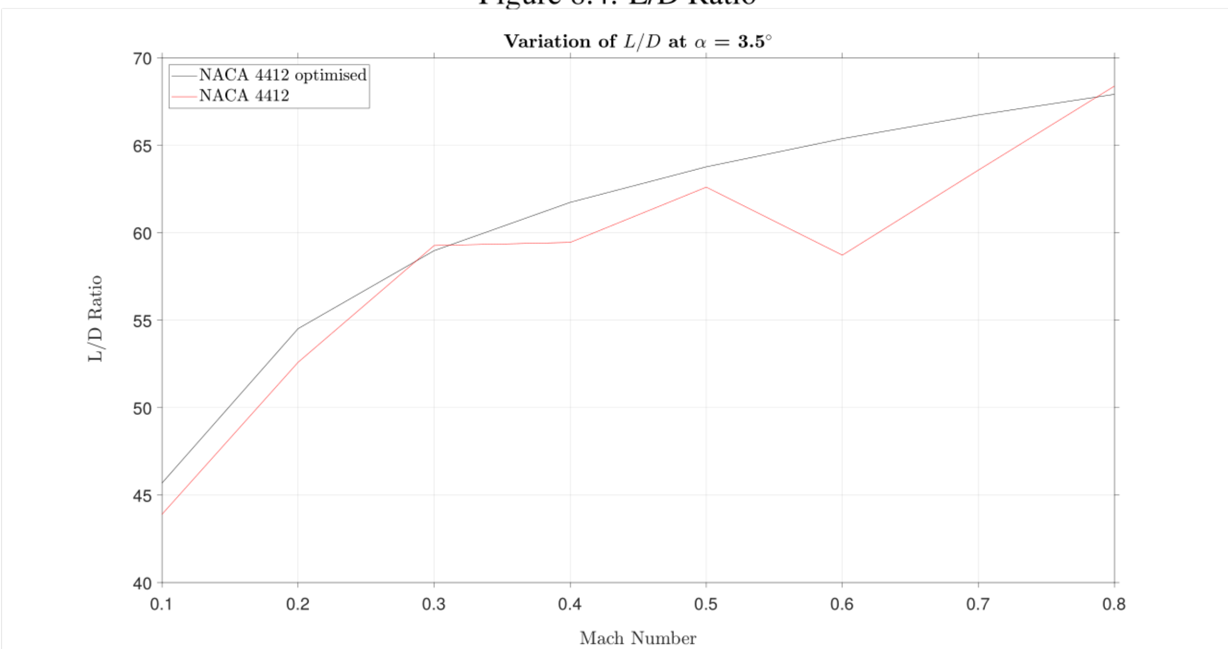


Figure 8.5: Lift

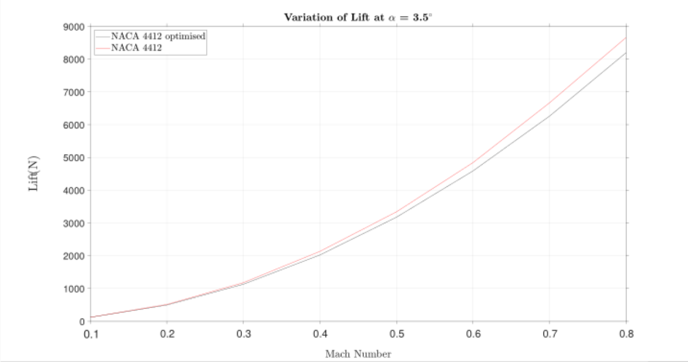
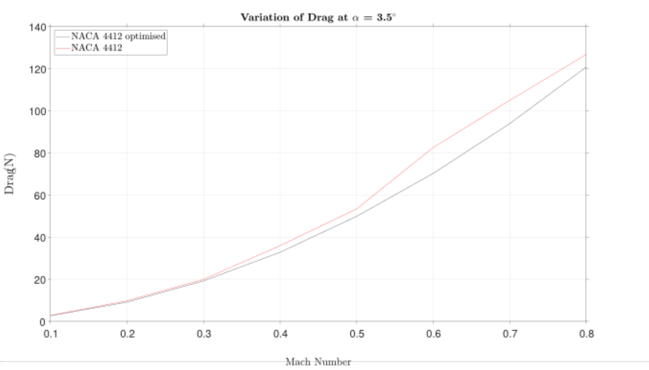


Figure 8.6: Drag



9 Conclusions

The results obtained at the end of the study conform to the problem parameters, and satisfactory results have been achieved.

10 References

[1] Albert E. Von Doenhoff and Ira Abott, "*Theory of Wing Sections: Including a Summary of Airfoil Data*", 1949

[2] Brenda M. Kulfan, "*A Universal Parametric Geometry Representation Method - "CST"* ", June 18, 2012

[3] Pramudita Satria Palar, "*Short note on CST method for Airfoil Parameterization (MATLAB Code included)*", https://pramsatriapalar.wordpress.com/2013/06/15/short-note-on-cst-method-for-airfoil-parameterization-matlab-code-included/, June 15, 2013

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