

# Maslen's Method: An Overview

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## 1 Introduction

The Maslen's method is a theory based on the assumption of a thin shock layer. It is a comparatively simple method and has frequent application even today for the approximate analysis of hypersonic inviscid shock layers. Moreover, Maslen's method gives results for the flow field over blunt as well as slender bodies.

## 2 MATLAB Code

```
1 % HSA project
2 clear all
3 clc
4 format long;
5 %% free stream properties
6 Y=1.4; % free stream specific heat ratio (gamma)
7 r=287; % gas constant
8 M=10; %mach no. is consider to be infinity
9 P=10000; % free stream pressure in pa
10 T=300; % free stream temperature in k
11 rho=P/(287*T); % free stream density in kg/m^3
12 cp=Y*r/(Y-1); %cp
13 V=M*(Y*r*T)^0.5; % velocity
14 %% assuming shock shape and parameters
15 R=1; %nose radius
16 d=(0.386*exp(4.67/M^2))*R; %d-shock standoff distance
17 Rc=(1.386*exp(1.8/(10-1)^0.1))*R; %Rc- shock radius of curvature at vertex
18 b=pi/20; % b= asymptotic shock wave angle
19 y=linspace(0,5,50);
20 x=-(R+d-Rc*(cot(b)^2)*(((1+(y.^2*tan(b)^2)/Rc^3)).^.85)-1));
21 for i=1: numel(y)-1
22     dx(i)=x(i+1)-x(i);
23     dy(i)=y(i+1)-y(i);
24     sl(i)=dy(i)/dx(i);% sl=slop
25 end
26 beta=atan(sl);
27 n=numel(sl);
28 for i=1:n-1
29     dX(i)=x(i+1)-x(i);
```

```

30 dY(i)=sl(i+1)-sl(i);
31 dsl(i)=dY(i)/dX(i);
32 end
33 betal=atan(dsl);
34 for i=1:numel(dsl)
35     if abs(dsl(1,i))>0
36         s(i)=(1+(sl(i))^2)^1.5/abs(dsl(i));
37     else
38         break
39     end
40 end
41 % shock defn ends
42 %Properties calculated just after the shock using approximated oblique %shock relations
43 P1=(1+((2*Y/(Y+1))*(M^2(sin(beta)).^2)-1))*P;
44 rho1=(1+((Y+1)M^2(sin(beta)).^2)/(((Y-1)M^2(sin(beta)).^2)+2))*rho; % density ratio
45 T1=((P1/P).*(rho./rho1))*T; %temperature ratio
46 u1=V*cos(beta);
47 P0=P*((1+(Y-1)*M^2/2)^(Y/(Y-1)));
48 T0=T*(1+(Y-1)*M^2/2);
49 Mn=M*sin(beta);
50 Mn1=[(Mn.^2+(2/(Y-1)))/([2*Y*Mn.^2/(Y-1)]-1)].^0.5;
51 theta=atan(2*cot(beta).[(M^1(sin(beta)).^2)-1]/((M^2*(Y+cos(2*beta))))+2));
52 M1=Mn1./sin(beta-theta);
53 P01=P1.*((1+(Y-1)*M1.^2/2).^(Y/(Y-1)));
54 dels=-8.314.*log(P01./P0);
55 q1= 0.5*rho1*V^2;
56 W=rho.*V.*y;
57 %% iterartive meslon's method
58 for i=1:numel(s)
59     for j=1:i
60         if j==1
61             P2(i,j)=P1(i);
62             T2(i,j)=T1(i);
63             rho2(i,j)=P1(i)/(287*T1(i));
64             u2(i,j)=(2*cp*(T0-T1(i)))^0.5;
65         else
66             P2(i,j)=P2(i,j-1)+((u1(i)/s(i))*(W(i-1)-W(i)));
67             T2(i,j)=(exp((r*log(P2(i,j)/P2(i,j-1))-dels(i))/cp))*T1(i);
68             rho2(i,j)=P2(i,j)/(287*T2(i,j));
69             u2(i,j)=(2*cp*(T0-T2(i,j)))^0.5;
70         end
71         if j<i
72             dn(i,j)=(2*(W(i)-W(i-j)))/(rho1(i)*u1(i)+rho2(i,j)*u2(i,j));
73             ang(i,j)=pi/2-beta(i);
74             x2(i,j)=x(i)+dn(i,j)*cos(ang(i,j));
75             y2(i,j)=y(i)-dn(i,j)*sin(ang(i,j));
76         end
77     end
78 end
79 for i=1:47
80     xb(i)=x2(i+1,i);
81     yb(i)=y2(i+1,i);
82     Pb(i)=P2(i+1,i);
83 end
84 %% Plots
85 figure(1),plot(x,y,'.',xb,yb,'-');
86 title('Shock and body shape');
87 xlabel('x-cordinate');
88 ylabel('y-cocordinate');
89 legend('shock shape','Body');
90 figure(2),plot(xb(2:numel(xb)),Pb(2:numel(xb)),'--');
91 title('Pressure distribution');
92 xlabel('x-cordinate');
93 ylabel('Pressure (pa)');

```

### 3 Preliminary Results

Some preliminary results are given below,

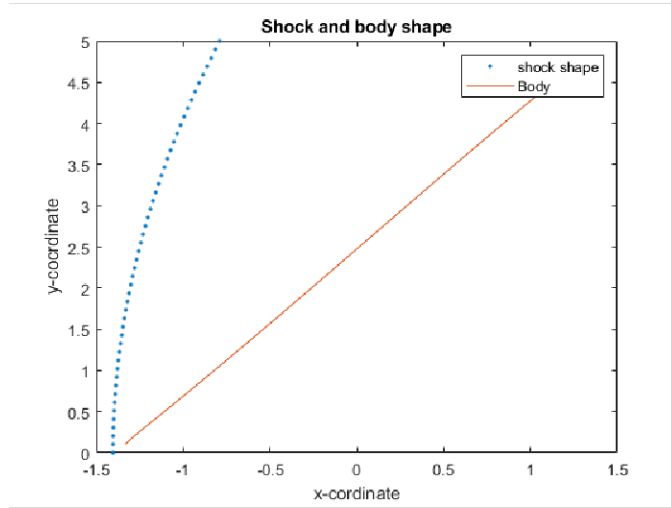


Figure 1: Shock and Body Shape

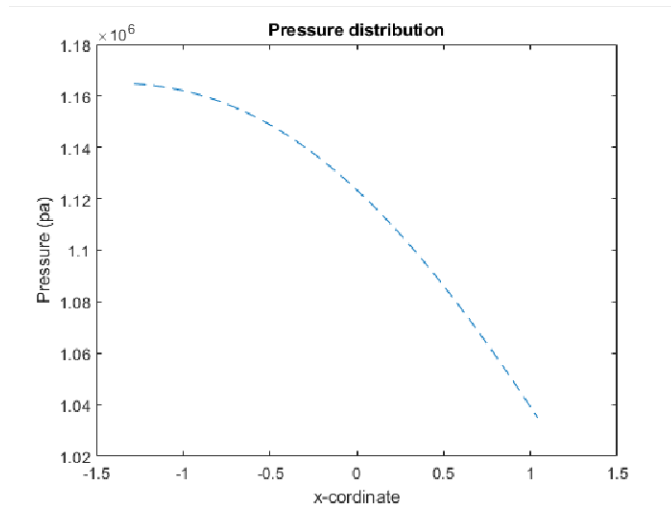


Figure 2: Pressure Distribution

### 4 Methodology

Consider the curvilinear coordinate system, where  $x$  and  $y$  respectively, are parallel and perpendicular to the shock, and  $u$  and  $v$  are corresponding components of velocity. For simplicity we will assume a two dimensional flow; however, Maslen’s method also applies to axisymmetric flow. Now we assume that the shock layer is thin and hence the streamlines are essentially parallel to the shock wave. In a streamline-based coordinate system, The momentum equation for the present coordinate system is given by,

$$\rho \frac{u^2}{R} = \frac{\partial p}{\partial y} \tag{1}$$

Where  $R$  is the local streamline radius of curvature. For thin shock-layer assumptions,

$$\rho \frac{u^2}{R_S} = \frac{\partial p}{\partial y} \quad (2)$$

Where  $R_S$  is the local shock radius of curvature. Further simplifying,

$$\rho \frac{u^2}{R_S} = \left( \frac{\partial p}{\partial \psi} \right) \rho u \quad (3)$$

This leads to the expression,

$$\frac{\partial p}{\partial \psi} = \frac{u}{R_S} \quad (4)$$

According to the earlier assumptions made of thin shock layer, we can consider  $u \approx u_S$  where  $u_S$  is the the velocity just behind the shock. By this assumption we are re-asserting the assumption that all the streamlines are parallel to the shock, therefore,

$$\frac{\partial p}{\partial \psi} = \frac{u_S}{R_S} \quad (5)$$

We can integrate the above equation between a point in the shock layer where the value of the stream function as  $\psi$  and just behind the shock layer where  $\psi = \psi_S$ . Thus,

$$p(x, y) = p_S(x) + \frac{u_S(x)}{R_S(x)} [\psi - \psi_S(x)] \quad (6)$$

Using the above equation we can build an inverse method where a shock wave shape will be assumed for a body to solve the above equation and then obtain the shape and pressure distribution over the body. When the obtained body shape matches with the real shape then we can get the shock shape and pressure distribution. The procedure described by Maslen can be summarized as,

- Assume a shock-wave shape. In a sense, Maslen's method is an inverse method, where a shock wave is assumed and the body that supports thin shock is calculated.
- Hence, all flow quantities are known at a point just behind the shock, from the oblique shock relations. The value for  $\psi_1$  is known from,

$$\psi = \rho_\infty V_\infty h \quad (7)$$

- Choose a value of  $\psi_2$ , where  $0 < \psi_2 < \psi_1$ . This identifies a point 2 inside the flowfield along the  $y$  axis, where  $\psi = \psi_2$ . (The precise value of the physical coordinate  $y_2$  will be found in a subsequent step).
- Calculate the pressure at point 2 from the equation,

$$p_2 = p_1 + \frac{u_1}{R_S} (\psi_2 - \psi_1) \quad (8)$$

- The entropy at point 2,  $s_2$  is known because the streamline at point 2, corresponding to  $\psi = \psi_2$ , has come through that point on the shock wave, point 2', where  $\psi_{2'} = \psi_2$ , and where

$$\psi_{2'} = \psi_2 = \rho_\infty V_\infty h_2 \quad (9)$$

or,

$$h_2 = \frac{\psi_2}{\rho_\infty V_\infty} \quad (10)$$

Therefore,  $h_2$  is obtained from the above equation, which locates point 2' on the shock. In turn,  $s_{2'}$  is known from the oblique shock relations, and because the flow is isentropic along any given streamline  $s_2 = s_{2'}$ . Calculating the enthalpy  $h_2$  and density  $\rho_2$  from the thermodynamics equations of state,

$$h_2 = h(s_2, p_2) \quad \rho_2 = \rho(s_2, p_2) \quad (11)$$

- Calculating the velocity at point 2 from the adiabatic equation (total enthalpy is constant),

$$h_o = h_\infty + \frac{v_\infty^2}{2} \quad (12)$$

Where  $h_o$  is the total enthalpy, which is constant throughout the adiabatic flow-field. In turn,

$$h_o = h_2 + \frac{u_2^2}{2} \quad (13)$$

Thus,

$$u_2 = \sqrt{2(h_o - h_2)} \quad (14)$$

- All of the flow quantities are now known at point 2. Now repeat the preceding steps for all points along the y axis between the shock (point 1) and the body (point 3). The body surface is defined by  $\psi = 0$ .
- The physical coordinates y, which corresponds to a particular value of  $\psi$ , can now be found by integrating the definitions of the stream function (which is essentially the continuity equation). Since,

$$\frac{d\psi}{dy} = \rho u \quad (15)$$

Then,

$$y = \int_{\psi}^{\psi_s} \frac{d\psi}{\rho u} \quad (16)$$

Where  $\rho$  and  $u$  are known as a function of  $\psi$  from the preceding steps. This also locates the body coordinate, where

$$y_b = \int_0^{\psi_s} \frac{d\psi}{\rho u} \quad (17)$$

- This procedure is repeated for any desired number of points along the specified shock wave, hence generating the flowfield and body shape which supports that particular shock.